A Game Theoretic Model for User Preference-Aware Resource Pricing in Wireless Mobile Networks

Undergraduate Research Report

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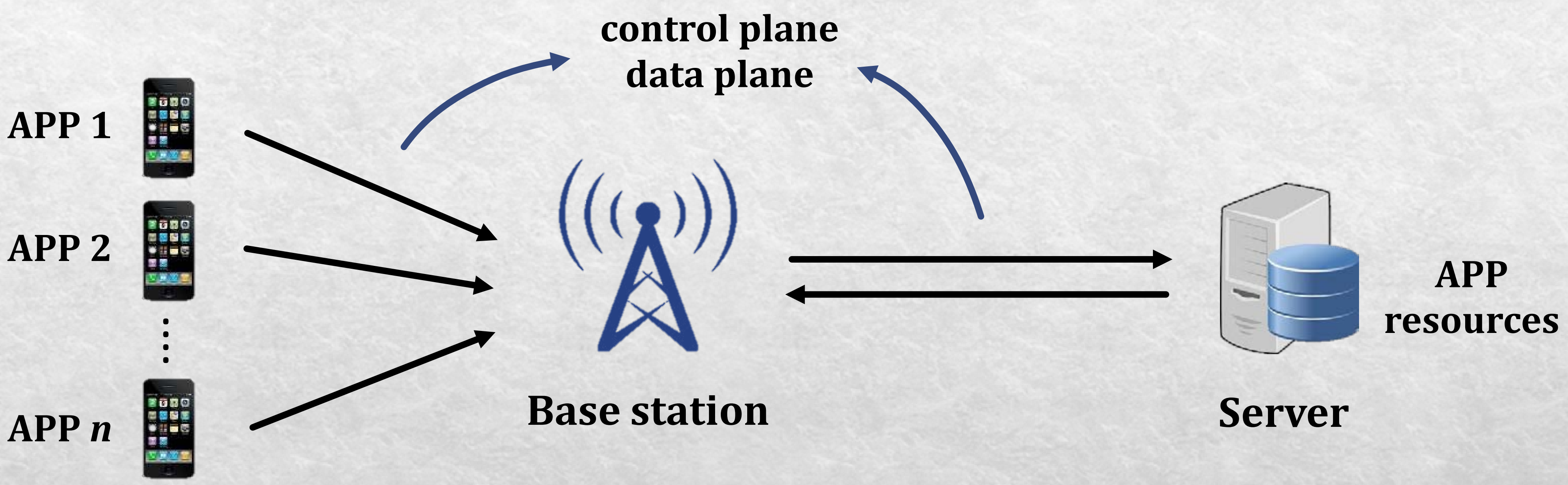
Outline

- Game Model
- Solving Stackelberg Game
- Results

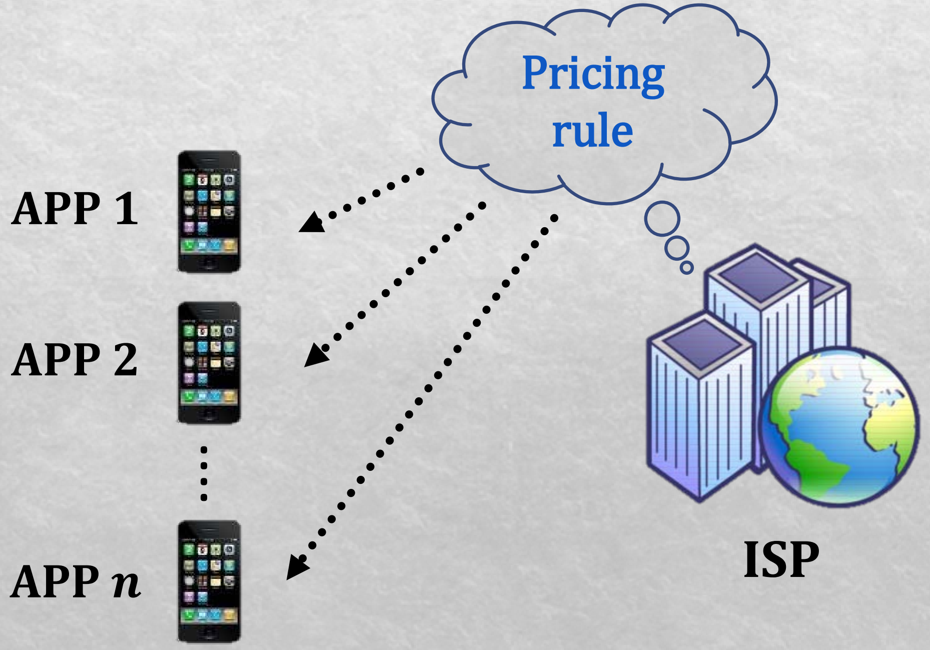
Game Model

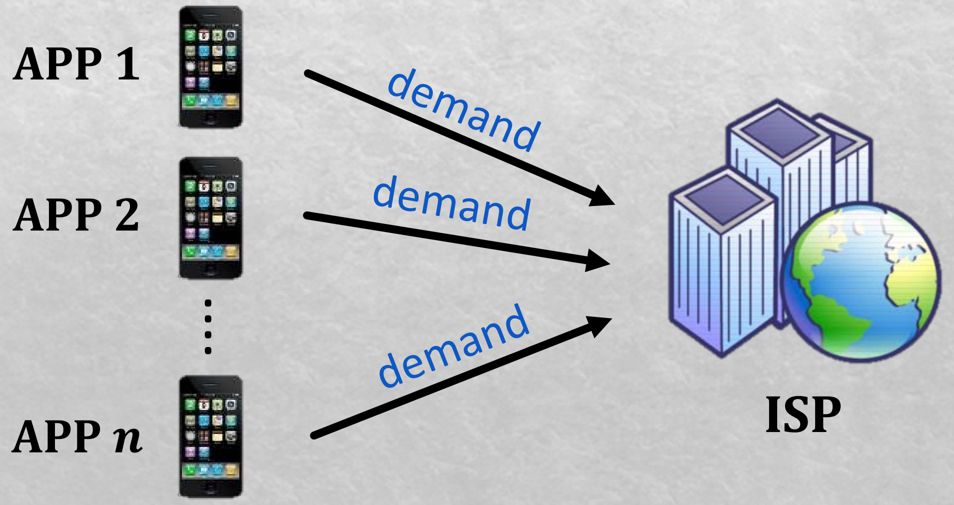


- Players: APP 1, APP 2, ... , APP n and ISP
- Resources: $\boldsymbol{R_C}$ (Control), $\boldsymbol{R_D}$ (Data), $\boldsymbol{R_A}$ (APP)

Non-cooperative Stackelberg Game

- First the ISP announces a pricing rule for the users.
- Then the users request for resource based on their demand.





Utility Functions

For user i:

$$\boldsymbol{U_i = min\left\{w_i log\left(1 + \frac{x_{C,i}}{a_i}\right), w_i log\left(1 + \frac{x_{D,i}}{b_i}\right), w_i log\left(1 + \frac{x_{A,i}}{c_i}\right)\right\} - (p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i})}$$

For the ISP :

$$\boldsymbol{U_{ISP} = \sum_i (p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i})}$$

$\boldsymbol{p_C, p_D, p_A}$: per data price $\boldsymbol{x_{C,i}, x_{D,i}, x_{A,i}}$: demand of user i

$\boldsymbol{w_i}$: user types $\boldsymbol{a_i, b_i, c_i}$: user prefernce, where $a_i + b_i + c_i = 1$

Two-stage Stackelberg Game

$$\begin{cases} U_i = min\left\{w_i log\left(1+\frac{x_{C,i}}{a_i}\right), w_i log\left(1+\frac{x_{D,i}}{b_i}\right), w_i log\left(1+\frac{x_{A,i}}{c_i}\right)\right\} \\ \qquad -(p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i}) \\ U_{ISP} = \sum_i (p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i}) \end{cases}$$

Stage 1 – Given p_C, p_D, p_A, maximize U_i

Stage 2 – Given $\left(x^*_{C,i}, x^*_{D,i}, x^*_{A,i}\right) = f(p_C, p_D, p_A)$, maximize U_{ISP}

Solving Stackelberg Game

Step 1 - Given $\boldsymbol{p_C}, \boldsymbol{p_D}, \boldsymbol{p_A}$, maximize $\boldsymbol{U_i}$

$$\boldsymbol{U_i = min\left\{w_i log\left(1 + \frac{x_{C,i}}{a_i}\right), w_i log\left(1 + \frac{x_{D,i}}{b_i}\right), w_i log\left(1 + \frac{x_{A,i}}{c_i}\right)\right\} - \left(p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i}\right)}$$

(1) From the minimal function part, we get

$$w_i log\left(1 + \frac{x_{C,i}}{a_i}\right) = w_i log\left(1 + \frac{x_{D,i}}{b_i}\right) = w_i log\left(1 + \frac{x_{A,i}}{c_i}\right) \Rightarrow \boldsymbol{\frac{x_{C,i}}{a_i} = \frac{x_{D,i}}{b_i} = \frac{x_{A,i}}{c_i}}$$

Solving Stackelberg Game

Step 1 - Given $\boldsymbol{p_C}, \boldsymbol{p_D}, \boldsymbol{p_A}$, maximize $\boldsymbol{U_i}$

$$\boldsymbol{U_i = w_i log\left(1 + \frac{x_{C,i}}{a_i}\right) - (p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i})}$$

(2) Now we maximize U_i

$$\frac{\partial U_{\text{User } i}}{\partial x_{C,i}} = \frac{\partial U_{\text{User } i}}{\partial x_{D,i}} = \frac{\partial U_{\text{User } i}}{\partial x_{A,i}} = 0 \Rightarrow \boldsymbol{(x_{C,i}, x_{D,i}, x_{A,i}) = (a_i k_i, b_i k_i, c_i k_i)}$$

$$\text{where } \boldsymbol{k_i = \frac{w_i}{a_i p_C + b_i p_D + c_i p_A} - 1}$$

Solving Stackelberg Game

Step 2 - Given $\left(x_{C,i}^*, x_{D,i}^*, x_{A,i}^*\right) = f(p_C, p_D, p_A)$, maximize U_{ISP}

$$\boldsymbol{U}_{\mathbf{ISP}} = \sum_i \left(p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i}\right)$$

$$\Rightarrow \frac{\partial U_{\mathrm{ISP}}}{\partial p_C} < 0\ \forall p_C > 0, \qquad \frac{\partial U_{\mathrm{ISP}}}{\partial p_D} < 0\ \forall p_D > 0, \qquad \frac{\partial U_{\mathrm{ISP}}}{\partial p_A} < 0\ \forall p_A > 0$$

<u>**Capacity Limitation:**</u>

$$\sum_i \left(\boldsymbol{x}_{C,i} + \boldsymbol{x}_{D,i} + \boldsymbol{x}_{A,i}\right) \leq \boldsymbol{C} \Leftrightarrow \sum_i \left(\frac{w_i}{a_i p_C + b_i p_D + c_i p_A}\right) \leq C + N$$

Solving Stackelberg Game

Step 2 - Given $(x_{C,i}^{*}, x_{D,i}^{*}, x_{A,i}^{*}) = f(p_C, p_D, p_A)$, maximize U_{ISP}

Goal:

$$\boldsymbol{maxmize\ U_{ISP} = \sum_i (p_C x_{C,i} + p_D x_{D,i} + p_A x_{A,i})}$$

$$\boldsymbol{with \sum_i \left(\frac{w_i}{a_i p_C + b_i p_D + c_i p_A}\right) \leq C + N}$$

Solving Stackelberg Game

Step 2 - Given $\left(x_{C,i}^{*}, x_{D,i}^{*}, x_{A,i}^{*}\right) = f(p_C, p_D, p_A)$, maximize U_{ISP}

Capacity Limitation Plane: an approximated border of feasible region

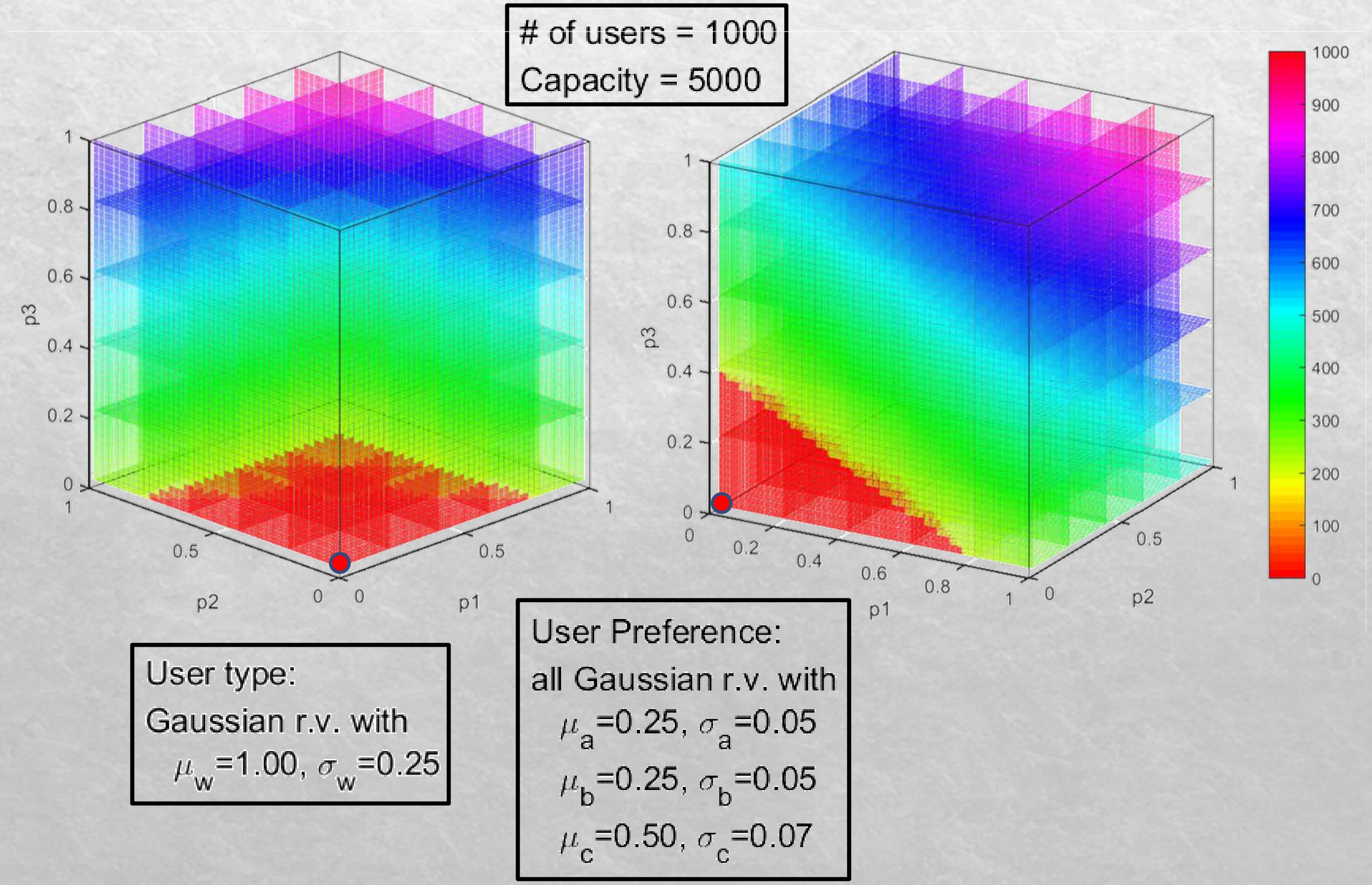
$$\sum_i \left(\frac{\boldsymbol{w_i}}{\boldsymbol{a_i p_C + b_i p_D + c_i p_A}} \right) = \boldsymbol{C + N}$$

assuming small variance on $\boldsymbol{a_i}, \boldsymbol{b_i}, \boldsymbol{c_i}$, we get

$$\Rightarrow \boldsymbol{\mu_a p_C + \mu_b p_D + \mu_c p_A = \frac{\sum w_i}{C + N}}$$

Solving Stackelberg Game

Step 2 - Given $\left(x_{C,i}^{*}, x_{D,i}^{*}, x_{A,i}^{*}\right) = f(p_C, p_D, p_A)$, maximize U_{ISP}



- Generally, the utility of ISP increase as the per data price goes down.
- The maximum revenue of ISP can be achieved around the capacity limitation plane: $\sum_i \left(x_{C,i} + x_{D,i} + x_{A,i}\right) = C$.

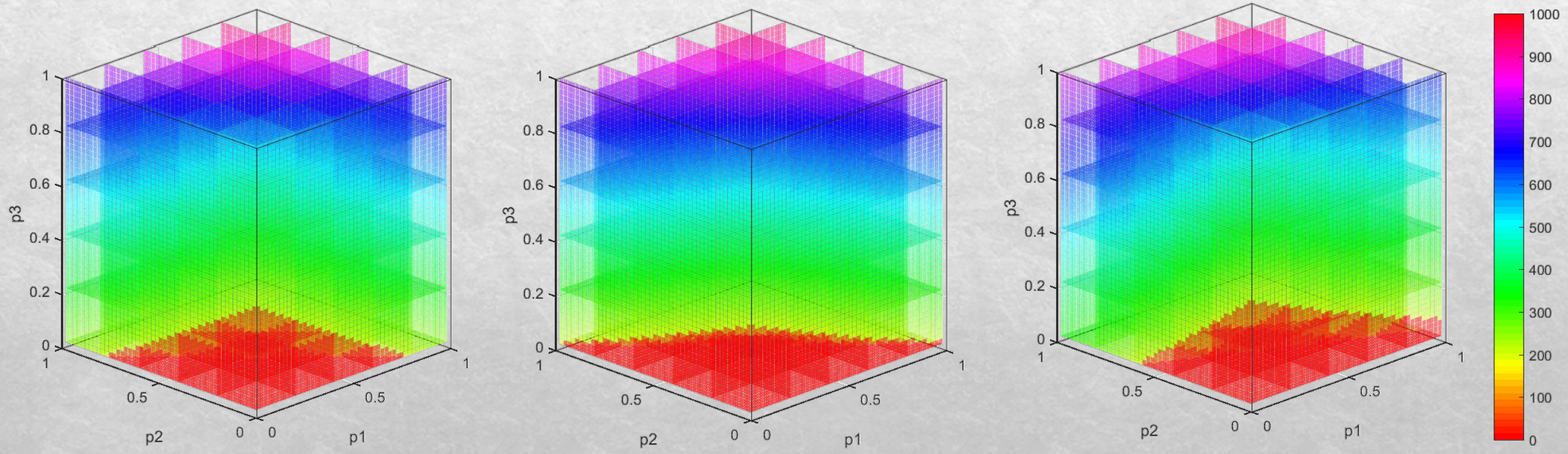
Solving Stackelberg Game

The limitation plane's direction changes with different combinations of user preference.

Effect of user preference

of users = 1000
Capacity = 5000

User type:
Gaussian r.v. with
μ_w=1.00, σ_w=0.25



$(\mu_a, \mu_b, \mu_c) = (0.25, 0.25, 0.5)$ $(\mu_a, \mu_b, \mu_c) = (0.2, 0.2, 0.6)$ $(\mu_a, \mu_b, \mu_c) = (0.17, 0.33, 0.5)$

Solving Stackelberg Game

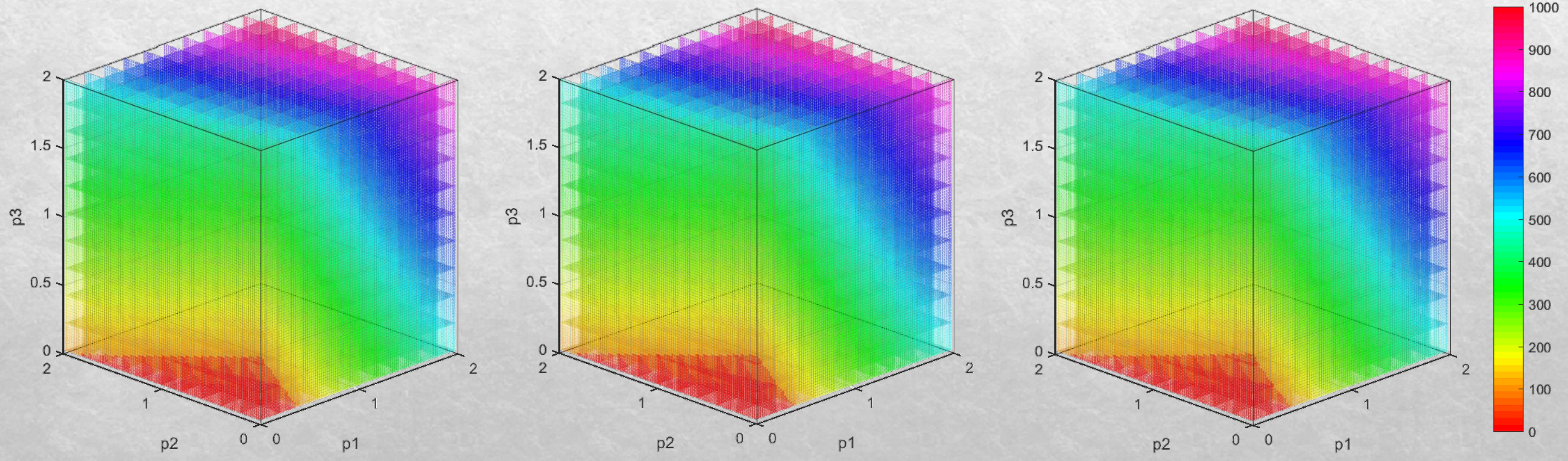
The border of feasible region becomes rough when the variance of user type increase.

Effect of user type variance

of users = 1000
Capacity = 5000

$\sigma_a = \sigma_c = 0.01$
$\sigma_b = 0.01$

$(\mu_a, \mu_b, \mu_c) = (0.5, 0.1, 0.4)$



$\sigma_a = \sigma_c = 0.01$
$\sigma_b = 0.01$

$\sigma_a = \sigma_c = 0.05$
$\sigma_b = 0.01$

$\sigma_a = \sigma_c = 0.10$
$\sigma_b = 0.01$

Solving Stackelberg Game

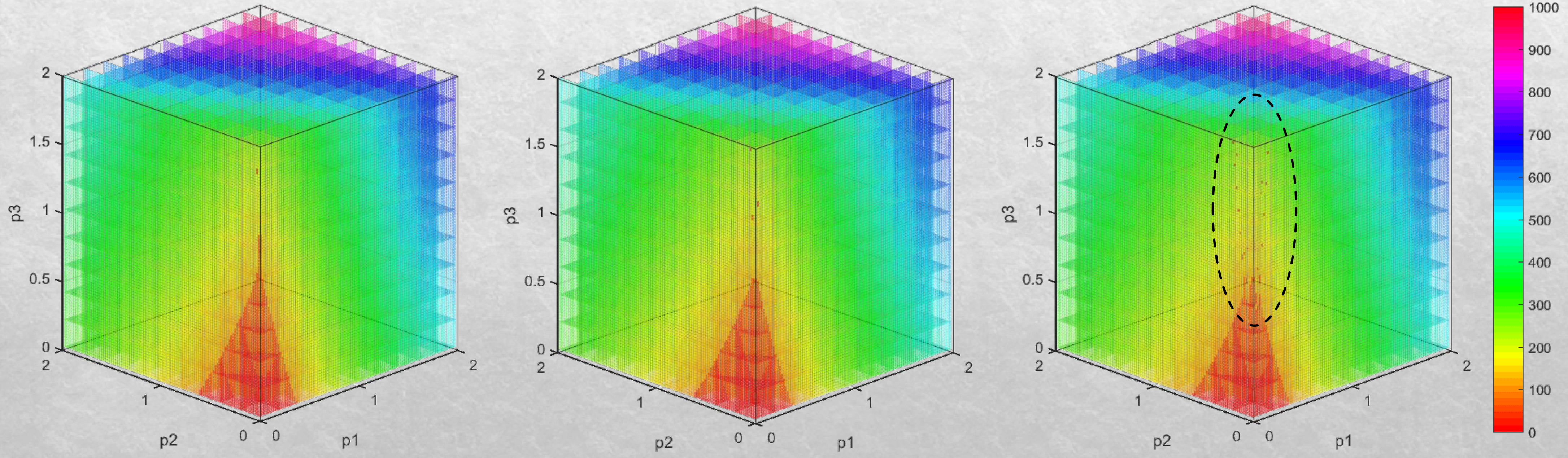
The border of feasible region becomes rough when the variance of user type increase.

Effect of user type variance

of users = 1000
Capacity = 5000

User type:
Gaussian r.v. with
μ_w=1.00, ~~σ_w=0.25~~

$(\mu_a, \mu_b, \mu_c) = (0.5, 0.3, 0.2)$
$(\sigma_a, \sigma_b, \sigma_c) = (0.05, 0.05, 0.07)$



$\boldsymbol{\sigma_w = 0.05}$ $\boldsymbol{\sigma_w = 0.15}$ $\boldsymbol{\sigma_w = 0.25}$

Results

- The optimal combination of (p_C, p_D, p_A) that results in the maximum revenue of the ISP is around **Capacity Limitation Plane**:

$$\boldsymbol{\mu_a p_C + \mu_b p_D + \mu_c p_A = \frac{\sum w_i}{C + N}}$$

- Picking any combination of (p_C, p_D, p_A) on **Capacity Limitation Plane** results in an approximately optimal utility of the ISP.

- If we want to simplify the pricing rule, we can simply choose

$$\boldsymbol{p_C = p_D = p_A = \frac{\sum w_i}{3\mu(C + N)}}$$

Thank you for your attention!

Appendix

Lemma

Given a random variable X with mean μ_x and variance σ_x,

$$\mathrm{E}[(x-\mu_x)^n] \ll {\mu_x}^n \; \forall \, n > 2 \quad \Rightarrow \quad \mathrm{E}\left[\frac{1}{X}\right] \sim \frac{1}{\mu_x} + \frac{\sigma_x^2}{2\mu_x^2}$$

Proof:

$$g(X) = g(\mu_x + X - \mu_x) = g(\mu_x) + (X-\mu_x)g'(\mu_x) + \frac{(X-\mu_x)^2}{2!}g'(\mu_x) + \cdots$$

$$E[g(X)] = E[g(\mu_x)] + E[(X-\mu_x)g'(\mu_x)] + E\left[\frac{(X-\mu_x)^2}{2!}g'(\mu_x)\right] + \cdots$$

Appendix

Proof: (continued)

$$E[g(X)] = E[g(\mu_x)] + E[(X-\mu_x)g'(\mu_x)] + E\left[\frac{(X-\mu_x)^2}{2!}g'(\mu_x)\right] + \cdots$$

$$= g(\mu_x) + 0 + \frac{\sigma_x^2}{2}g''(\mu_x) + \frac{\mathrm{E}[(x-\mu_x)^n]}{n!}g^{(n-1)}(\mu_x)$$

$$= \ \mu_x + 0 + \frac{\sigma_x^2}{2}g''(\mu_x) + \frac{\mathrm{E}[(x-\mu_x)^n]}{n!}g^{(n-1)}(\mu_x)$$

$$= \ \mu_x + 0 + \frac{\sigma_x^2}{12\mu_x^3} + \frac{1}{n!}\frac{\mathrm{E}[(x-\mu_x)^n]}{(n-1)!\,{\mu_x}^n}$$

$$\sim \mu_x + 0 + \frac{\sigma_x^2}{12\mu_x^3} \quad by\ \mathrm{E}[(x-\mu_x)^n] \ll {\mu_x}^n$$

Appendix

In our model, consider a_i, b_i, c_i as random variables A, B ,C, respectively.

Let $\boldsymbol{x_i} = \boldsymbol{a_i p_C} + \boldsymbol{b_i p_D} + \boldsymbol{c_i p_A}$, then

$$\mu_x = \mu_a p_C + \mu_b p_D + \mu_c p_A, \qquad \sigma_x = \sigma_a p_C + \sigma_b p_D + \sigma_c p_A$$

If $\forall\, n > 2, \mathrm{E}[(x_i - \mu_x)^n] \ll {\mu_x}^n$, then we have

$$\mathrm{E}\left[\frac{1}{a_i p_C + b_i p_D + c_i p_A}\right] \sim \frac{1}{\mu_a p_C + \mu_b p_D + \mu_c p_A} + \frac{(\sigma_a p_C + \sigma_b p_D + \sigma_c p_A)^2}{2(\mu_a p_C + \mu_b p_D + \mu_c p_A)^2}$$

$$\underbrace{\sum_i \left(\frac{w_i}{a_i p_C + b_i p_D + c_i p_A}\right) = \frac{\sum w_i}{\mu_a p_C + \mu_b p_D + \mu_c p_A}} + \underbrace{\frac{(\sigma_a p_C + \sigma_b p_D + \sigma_c p_A)^2}{2(\mu_a p_C + \mu_b p_D + \mu_c p_A)^2} \sum w_i}$$

approximation used in deriving limitation plane

approximation error