2012 Intel ISEF / Physics and Astronomy(Team Project)

PH306

Reverse Ordering in Dynamical Two-Dimensional Hopper Flow

Hao-Wen Dong Chen-Chieh Ping

Contents

Abstract

We study the exit ordering of grains in gravity driven flow through two-dimensional hoppers of different hopper angle and outlet size with adjustable reclining angle. We observe a reverse ordering phenomenon such that grains entering the hopper at earlier times may not come out earlier. We record the entry order and exit order of the grains and calculate the degree of reverse ordering which is found to increase with increasing hopper angle, decreasing reclining angle, and increasing hopper outlet size. In order to find the mechanism of reverse ordering, we construct maps which register the entry order and exit order according to the position of the grain in the hopper before the flow. By comparing the exit order map and the entry order map we locate the regions where grains undergo reverse ordering. From the trajectories of the grains in the reverse ordering regions, we find that they take part in avalanches at the surface on their way to the exit. Hence, it is the dynamical process of surface avalanche that reverse the exit order of the grain when they flow out of the hopper. These results may be useful for special hopper design in agricultural and pharmaceutical industries to reduce or to enhance reverse ordering of materials for specific purposes.

Introduction

Queuing is a sign of modern civilization, which makes things be done in order. However, inside a granular flow, grains will be mixed in general. In some cases, the grains that enter the hopper at earlier times, however, will exit later, which is the phenomenon of reverse ordering. In this study, we quantified and investigated the reverse ordering phenomenon in a two-dimensional (2D) gravity-driven hopper flow, examined the effects of the reclining angle and the hopper angle, and analyzed the phenomenon in different scales.

Materials and methods

1. Setup

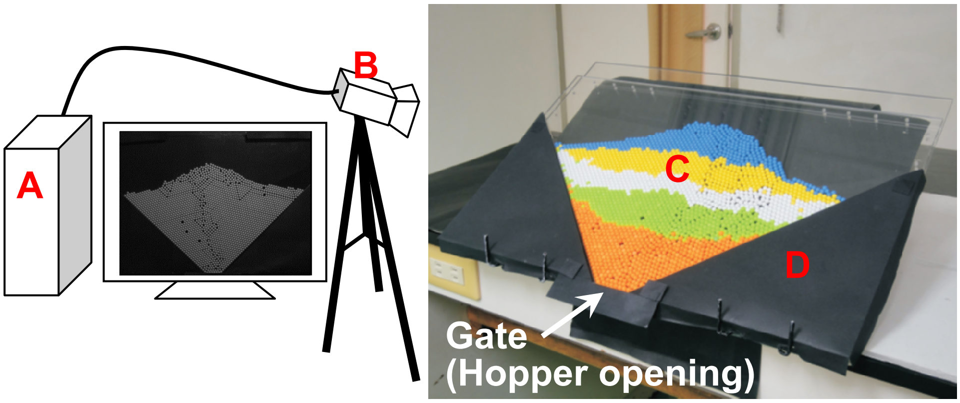

Fig. 1 Schematic of experimental setup

Fig. 2 The two-dimensional channel

A: PC

B: High speed camera (Dalsa GIGE camera, 300 fps) / Digital camera (Nikon S6000)

C: Grains (diameter=6mm; plastic BB bullets)

D: Two-dimensional hopper

E: Two-dimensional channel

2. Methods

(1) Qualitative experiments

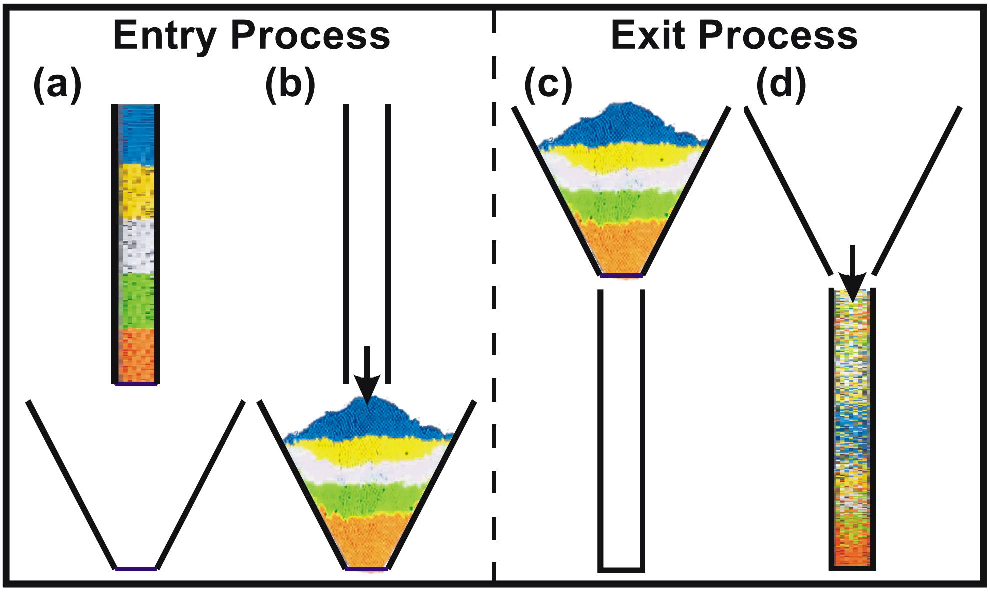

Fig. 3 Schematic of qualitative experimental setup

(a) Fill the grains with different colors into the 2D container in order and take a photo of it (Fig. 3 (a)).

(b) Set the digital camera ready.

(c) Fill up the hopper with grains through the 2D channel (to obtain a constant entry rate) (Fig. 3 (b)).

(d) Set the 2D channel at the opening of the hopper (Fig. 3 (c)).

(e) Start recording and open the gate.

(f) After the grains flow through the hopper and fill up the 2D container, close the gate and take a photo of the 2D container (Fig. 3 (d)).

(g) Repeat the experiment for at least three times.

(2) Quantitative experiments

(a) Start recording and fill up the hopper with grains through the 2D channel (to obtain a constant entry rate).

(b) Open the gate. After the grains drop out the hopper, close the gate and repeat the experiment for at least three times.

(c) Register the entry order and the exit order of the grains through a particle tracker program, and calculate the average value of the degree of reverse ordering.

3.Experimental parameters

The following parameters are used to investigate the reverse ordering phenomenon (Fig. 4):

(1) Reclining angle φ : 15º~90º

(2) Hopper angle θ : 30º~60º

(3) Coefficient of friction between side walls and grains μ : 0.52~1.27

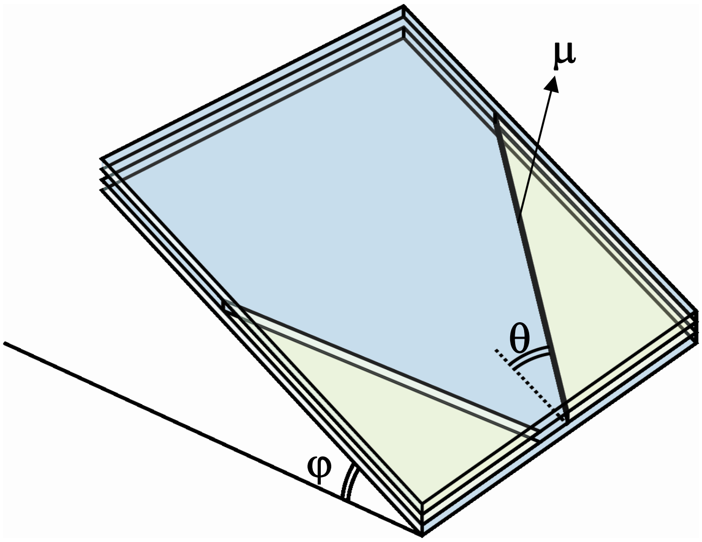

Fig. 4 Schematic of experimental setup

4. Qualitative Results

Fig. 5 The entry order at θ=30°, 60°

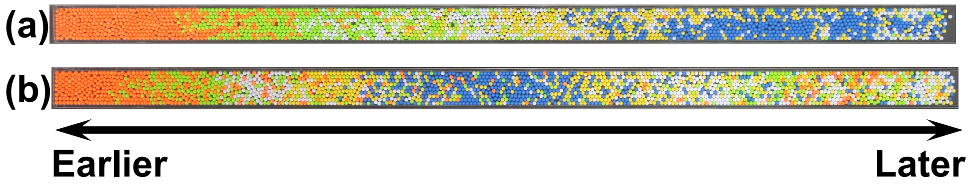

Fig. 6 The exit order at (a) θ=30° and (b) θ=60°

(1) The entry order of grains (Fig. 5) is different from the exit order (Fig. 6), and this means that the order of grains will change after they flow through the hopper. For example, when the hopper angle θ is 60°, the blue grains enter the hopper later; however, they come out earlier than some grains in different colors.

(2) The grains entering the hopper at the earlier times may not come out earlier, and we call such interesting phenomenon the **reverse ordering** of grains.

5. Quantitative analysis

(1) Particle tracker program

(a) A particle tracker program was used to identify the grains and find out the trajectories of them.

(b) We wrote a program that we can obtain the entry order and the exit order of grains (Appendix I).

(2) Quantitative methods

(a) To quantify the degree of reverse ordering of each grain i, we defined a quantity γ_i shown at the right side.

$$\gamma_i = I_i - O_i$$

(b) To quantify the degree of reverse ordering of an experiment, we defined another quantity Γ, which has been normalized and ranges 0 to 1.

$$\Gamma = \frac{1}{f(N)} \sqrt{\sum_{i=1}^{N} \gamma_i^2}$$

I_i: entry order of grain i
O_i: exit order of grain i
N: total number of grains
$f(N)$: normalizing factor

(c) A larger Γ means the reverse ordering is more obvious. When Γ=0, the exit order is the same as the entry order, which means there is no reverse ordering. When Γ=1, the order is completely reversed.

(c) The normalizing factor ($\sqrt{(N^3-N)/3}$) is the upper bond of $\sum_{i=1}^{N} \gamma_i^2$, obtained when the order is completely reversed (from 1~n to n~1).

(d) Here are examples of calculating Γ:

Perfect ordering

entry order		1	2	3	4	1999	2000
exit order	-)	1	2	3	4	1999	2000
γ_i		0	0	0	0	0	0

$$\Gamma = \frac{1}{\sqrt{2000^3 - 2000)/3}} \sqrt{0^2 + 0^2 ... + 0^2 + 0^2} = 0$$

Perfect reverse ordering

entry order		1	2	......	1999	2000
exit order	-)	2000	1999	......	2	1
γ_i		-1999	-1997	......	1997	1999

$$\Gamma = \frac{1}{\sqrt{(2000^3 - 2000)/3}} \sqrt{(-1999)^2 + ... + 1999^2} = 1$$

Results and discussion

1. Maps of grains' order

(1) We drew maps of the entry order and the exit order according to their initial locations (Fig. 7(a)) (Fig. 7(b)). Different colors represent the orders of grains. For instance, in the map of entry order (Fig. 7(a)), the blue grains will enter first while the red grains will enter later.

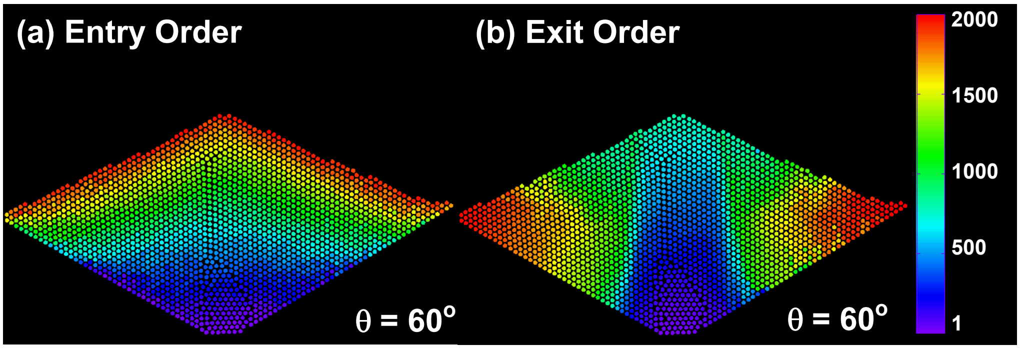

Fig. 7 Contour maps of grains' (a) entry order and (b) exit order

(2) The map of the entry order looks stratified horizontally, which means the grains arrange without too much mixing (Fig. 7(a)).

(3) In the map of the exit order, the grains with similar colors will exit the hopper together (Fig. 7(b)). The map looks stratified vertically, which is different from the map of entry order.

(4) To find out the region that grains undergo reverse ordering, we drew a map of the order difference (difference between entry order and exit order) (Fig. 8). We found the grains located at the central region will outpace other grains (γ_i > 0; red grains); however, a grain initially located at the side-wall region may be stranded on the side wall while flowing (γ_i < 0; blue grains).

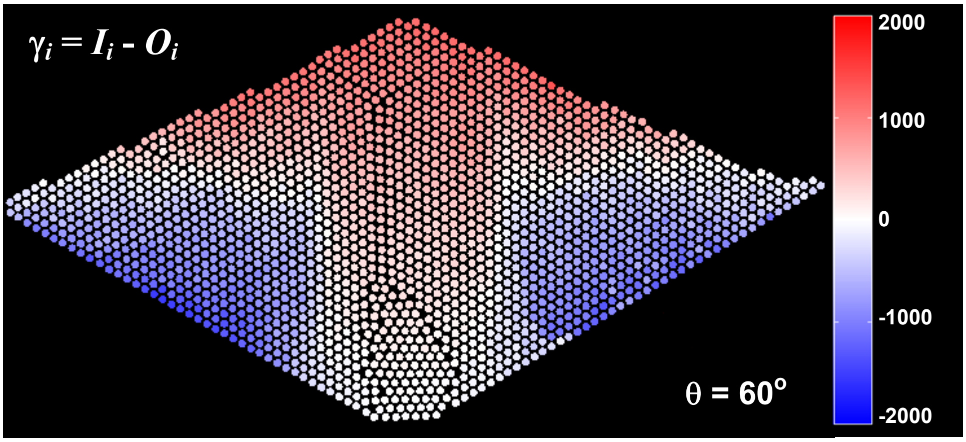

Fig. 8 Contour maps of the order difference γ_i

(5) When we changed the hopper angle, we found that the regions the grains undergo reverse ordering are always located at the central part and the side-wall part. When the hopper angle decreases, the colors will become lighter. This means that the degree of the reverse ordering becomes smaller (Fig. 9).

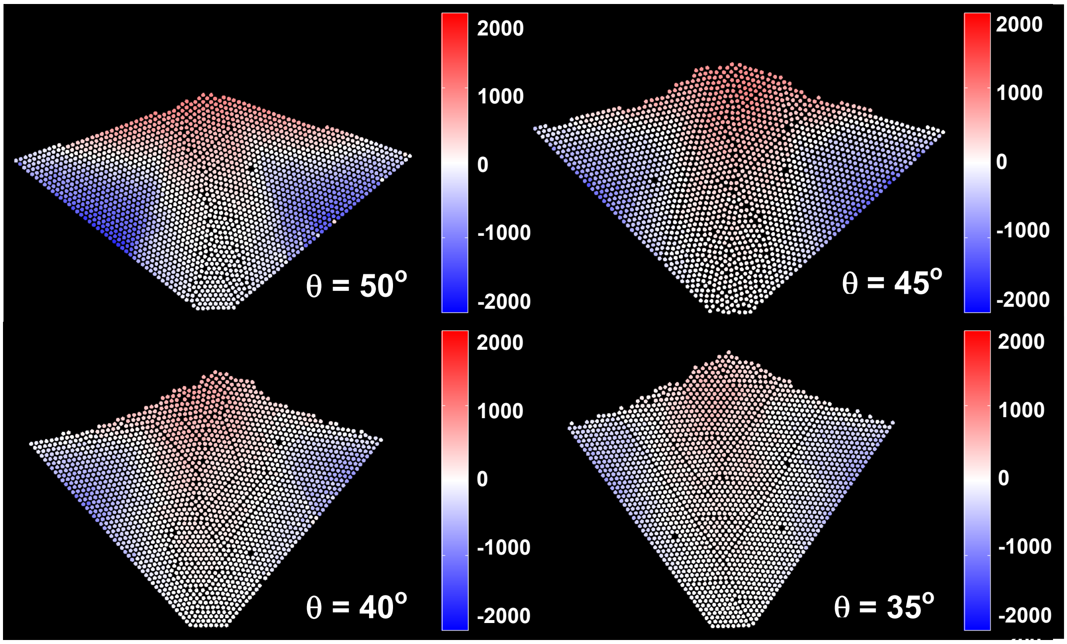

Fig. 9 Contour maps of the order difference γ_i with different hopper angles

2. Effects of angles on Γ

(1) Effects of reclining angle and hopper angle

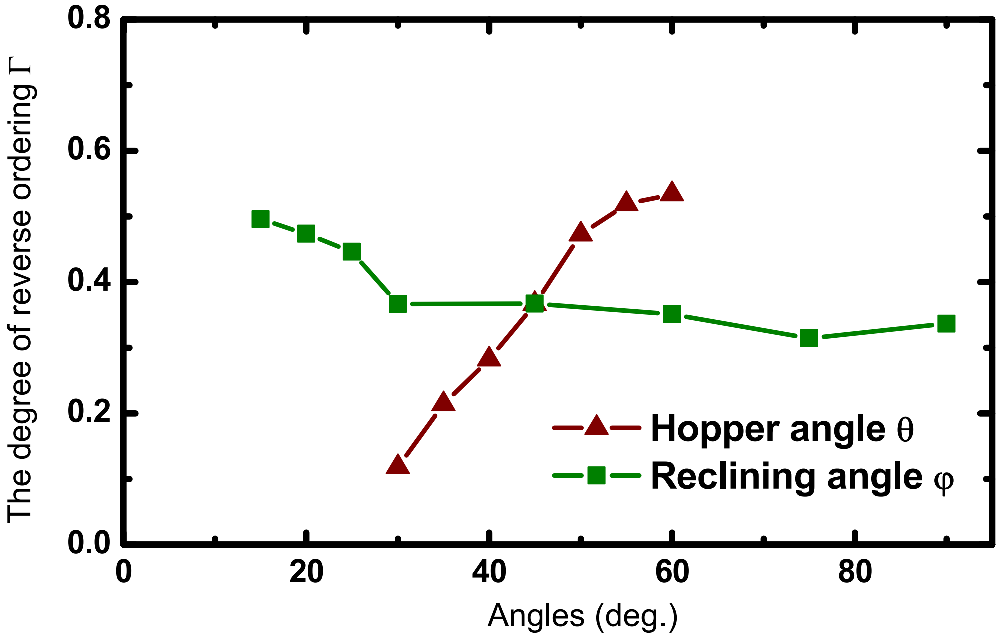

Fig. 10 Effects on the degree of reverse ordering Γ of reclining angle φ and hopper angle θ

(a) The degree of reverse ordering Γ increases with decreasing reclining angle φ (Fig. 10).

(b) The degree of reverse ordering Γ increases with increasing hopper angle θ (Fig. 10).

(c) We think a larger degree of reverse ordering results from a larger completion time difference between central flow and side-wall flow.

(d) Central flow starts right above the opening; side-wall flow starts on side walls.

(e) A larger reclining angle φ speeds up central flow and side-wall flow (Fig.11); however, the ratio between them doesn't change (Fig.12). Therefore, the completion time difference will decrease, which we think causes a smaller degree of reverse ordering Γ.

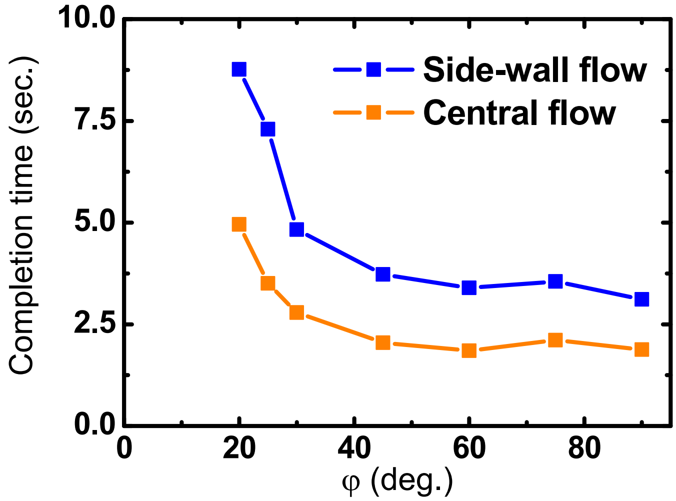

Fig. 11 The completion time for central flow and side-wall flow at different reclining angle φ

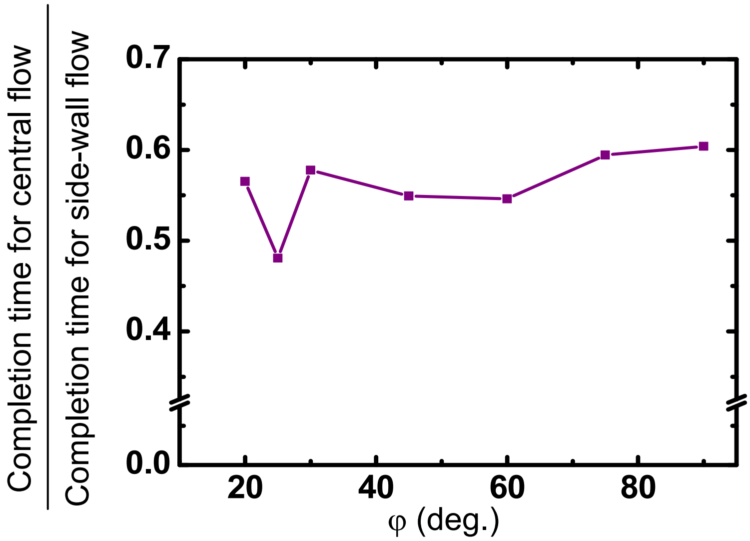

Fig. 12 The ratio of the completion time for central flow to that of side-wall flow at different reclining angle φ

(f) A larger hopper angle θ provides a larger frictional force for a grain on side wall, which slows down side-wall flow and increases the completion time difference (Fig. 13). We think this causes a larger degree of reverse ordering Γ.

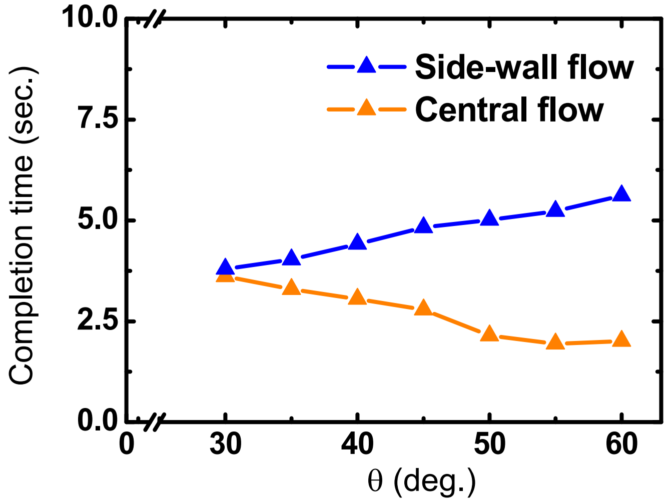

Fig. 13 The completion time for central flow and side-wall flow at different hopper angle θ

3. Discussion

(1) Completion time difference

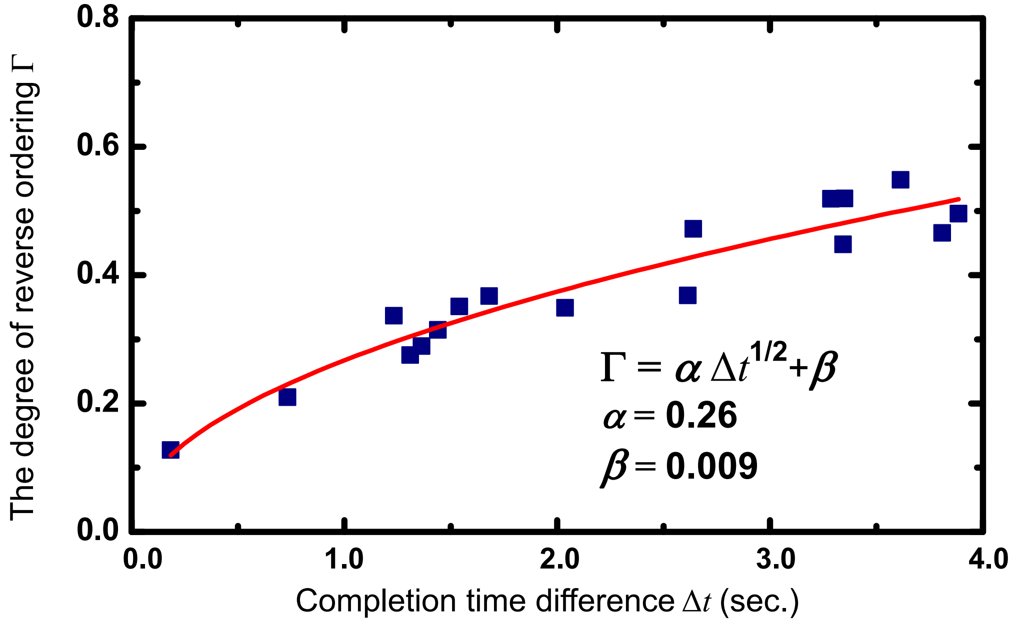

Fig. 14 Relation between the degree of reverse ordering and the completion time difference between central flow and side-wall flow

(a) To check our theory, we measured the completion time for central flow (t_c) and side-wall flow (t_s) and calculated their difference Δt (t_s-t_c).

(b) The degree of reverse ordering Γ is proportional to the square root of the completion time difference $\Delta t^{1/2}$ (Fig. 14).

(2) Reverse ordering and friction

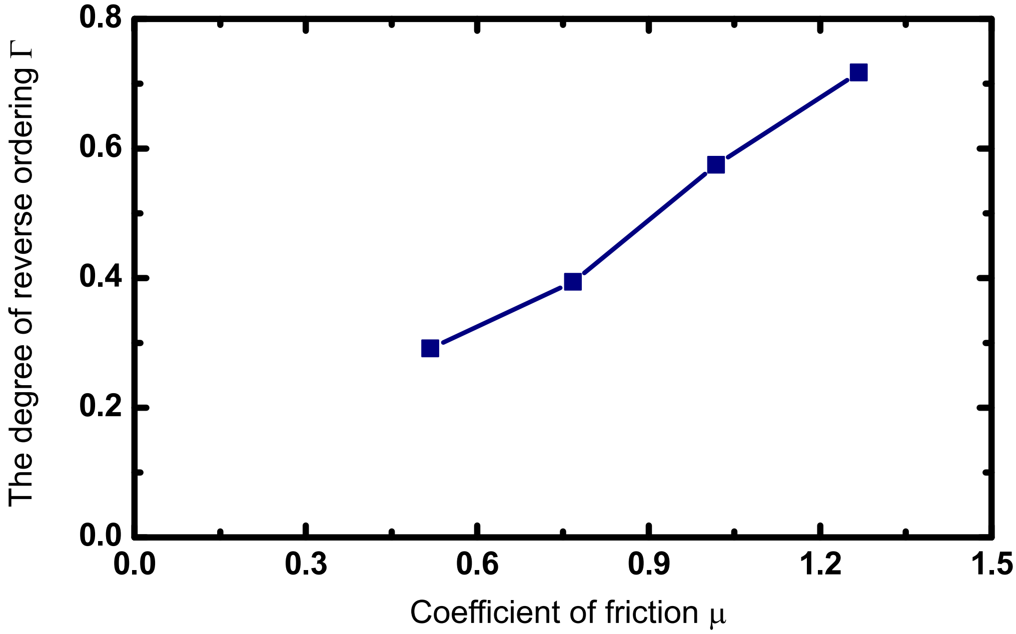

Fig. 15 Effects on the degree of reverse ordering Γ of the friction between grains and side walls

(a) We also check our theory by sticking anti-slip materials on side walls, which increase the friction between grains and side walls.

(b) The degree of reverse ordering Γ increases with increasing friction between grains and side walls (Fig. 15).

(c) A larger friction may slow down side-wall flow and increase completion time difference. This is consistent to our theory.

(3) Surface Avalanches

(a) Grains have different dynamical behavior, some of which will undergo avalanche at the surface of the flow, and we think this may influence the degree of reverse ordering.

(b) To identify the grains which undergo surface avalanches on the way to the opening of the hopper, we compared the behavior of the grains undergoing surface avalanches and those do not. Finally, we came up with the idea that the grains that undergo surface avalanches may have a larger horizontal velocity to outpace others.

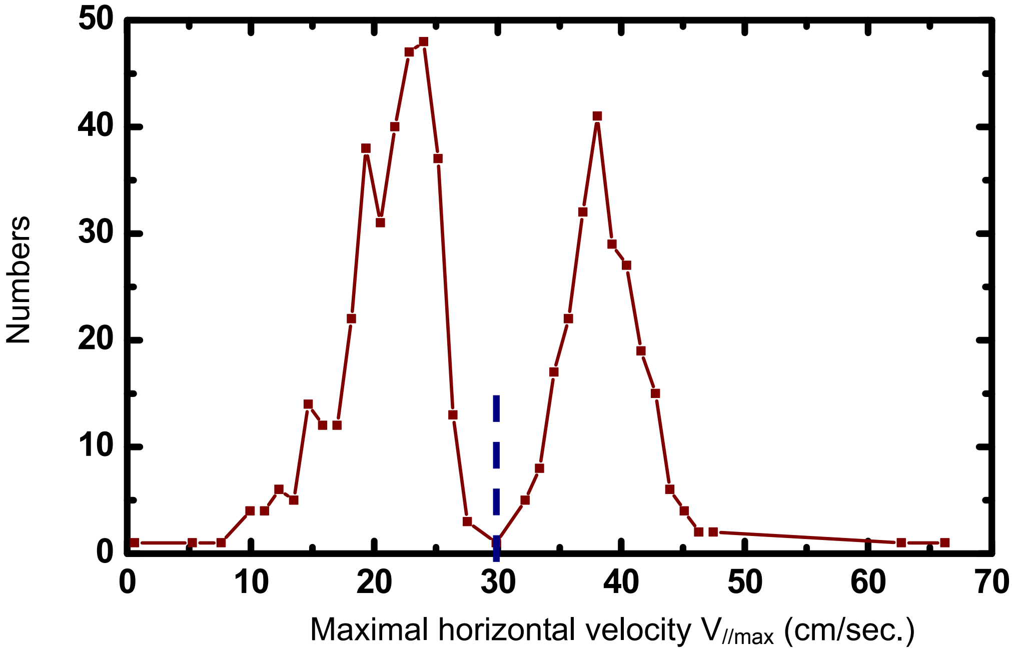

Fig. 16 Histogram of grain's maximal horizontal velocity $V_{//max}$ at θ=60°

(c) Therefore, we drew histograms of their maximal horizontal velocities $V_{//max}$ (Fig. 16). In some experiments, we found that there are two peaks, so we can set the threshold.

(d) We drew the grains whose maximal horizontal velocity is great than the threshold in red, and drew others in white (Fig. 17).

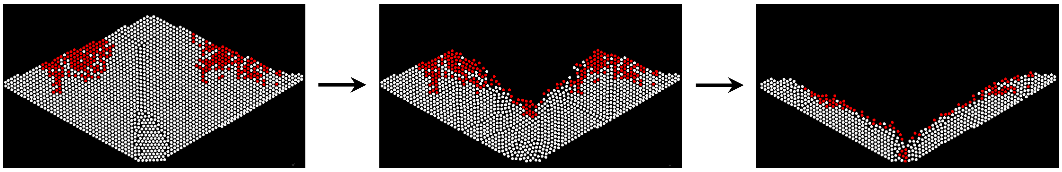

Fig. 17 Dynamical behavior of the grains that undergo surface avalanches

(e) We counted the number of grains that undergo surface avalanche through comparing their maximal horizontal velocity. We found that the degree of reverse ordering Γ increases with increasing number of the grains that undergo surface avalanche (Fig. 18). Thus, the avalanche at surface is another factor that results in the reverse ordering phenomenon.

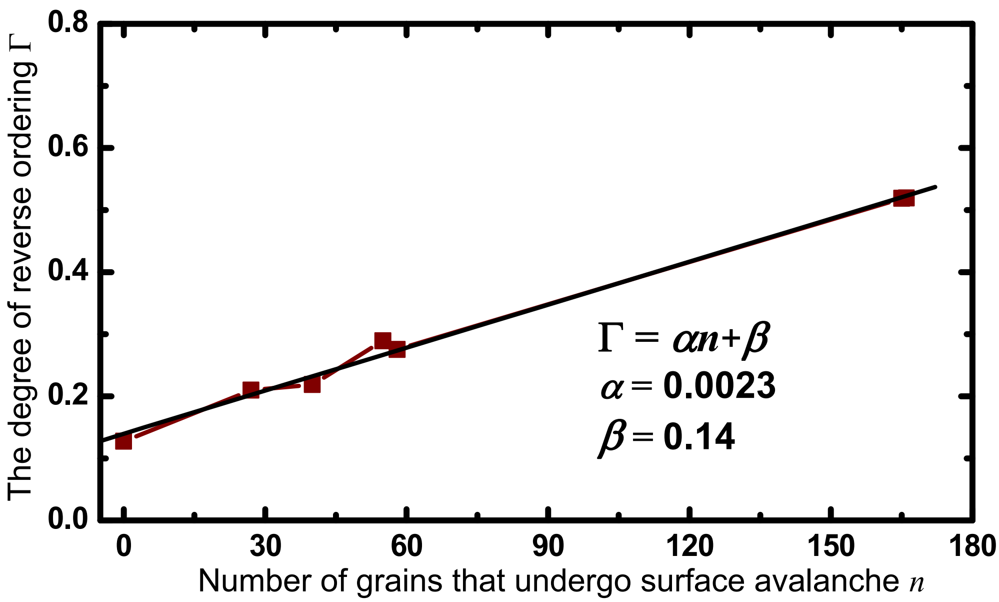

Fig. 18 Relation between the number of the grains undergoing surface avalanches n and the degree of reverse ordering

Conclusions

1. A reverse ordering phenomenon is observed.
2. Grains starting from the central part of the hopper will advance while those starting from the side-wall part of the hopper will retard.
3. The degree of reverse ordering can be quantified by Γ, the quantity we defined, which is sensitive to the hopper angle.
4. The completion time difference Δt between central flow and side-wall flow leads to the reverse ordering of grains, and Γ is proportional to $\Delta t^{1/2}$.
5. Behavior of surface avalanches influences the reverse ordering phenomenon.

Macroscopic	Mesoscopic	Microscopic
Effects on the degree of reverse ordering Γ	Completion time difference between central flow and side-wall flow	Behavior of Surface avalanches

Fig. 19 Reverse ordering in different scales

Applications

These results may be useful for special hopper design in agricultural and pharmaceutical industries. The reverse ordering may influence the dropping order of the grains while packing or Transporting, stranding and spoiling them, even affecting the freshness of the items. It may be applied on the packing of the items like millet, medicines, and ores.

Acknowledgement

We would like to thank

Prof. To, Institute of Physics, Academia Sinica

Prof. Lee, National Center University

Mr. Chien, Affiliated Senior High School of National Kaohsiung Normal University

References

[1]H. M. Jaeger, S. R. Nagel, and R. P. Behringer. Rev. Mod. Phys., 68, 1259 (1996).

[2]T. Nguyen, C. Brennen, and R. Sabersky. Journal of Applied Mechanics. 47, 6691 (1980).

[3]M. Hou, W. Chen, T. Zhang, K. Lu, and C. K. Chan. Phys. Rev., 91, 204301 (2003).

Appendix I

Program name: order.s

Usage: order.s xyti

Program purpose: Find out the entry order and the exit order of grains.

Input files: xyti

Format:

Column	1(x)	2(y)	3(t)	4(i)
Content	x position	y position	Frame numbers	Grains' ID

Input array should be sorted as rows with corresponding ID numbers in contiguous blocks, with the time variable a inside each block.

Output files: iIO

Format:

Column	1(i)	2(I)	3(O)
Content	Grains' ID	Entry Order	Exit Order

Output array is sorted with ID numbers a monotonically increasing function.

Side effects: ii (ID, entry frame number), io (ID, exit frame number)

Program Content:

```
#!/bin/bash
#script: order.s a
for f in $*; do
   cat $f | awk '{if ($4==i) {print $4, $3; i++}}' | sort > $f.ii
   max=`tail -1 $f | cut -d' ' -f4`
   tac $f | awk -v i=$max '{if ($4==i) {print $4, $3; i--}}' | sort > $f.io
   join $f.ii $f.io | sort +1 -n | awk '{print $1, NR, $3}' | sort +2 -n \
      | awk '{print $1, $2, NR}' | sort -n > $f.iIO
done
```